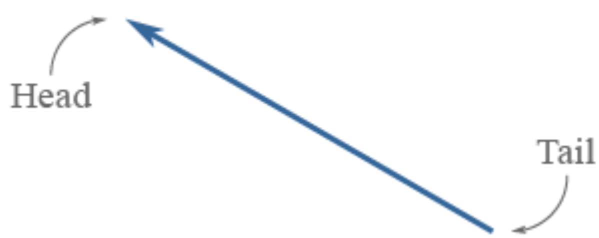
Chapter Summary

Chapter Summary: Vectors

Scalars are numbers that do not include information about direction or orientation. They may be integers or real numbers, and may have units. They may be positive or negative. Vectors have a magnitude, which is always a positive number that may be an integer or a real number, either with or without units, and a vector also has an orientation, or a direction. A vector is generally represented as an arrow or by either a dot or an x when the direction is along the line of sight as shown in the following two figures.



A vector is drawn from its tail to its head, which is at the tip of an arrowhead. The direction of a vector is indicated by the direction of the arrow. The magnitude of a vector is proportional to its length, the distance between its head and tail.

The solid dot represents a vector that is perpendicular to the plane of the image and directed out of the page, toward the observer. An $\times$ represents a vector that is perpendicular to the plane of the page and is directed into the page, away from the observer.

Scalar Multiplication

The magnitude of a vector can be changed by multiplying it by a scalar quantity. If the scalar has a positive value, the direction is unchanged. If the multiplication is by a negative scalar, the direction of the vector is reversed.

Cartesian Coordinates

When working with **Cartesian Coordinates**, each dimension is represented by its own number line as shown in the figure below.

- In one dimension, Cartesian coordinates require a single **coordinate axis** extending on one end towards negative infinity while the other end extends towards positive infinity. If we refer to this first axis as the x axis, then any point on this axis is an x **coordinate**. The unit vector $\hat{i}$ is in the $+x$-direction. In many cases, this is also referred to as the **horizontal axis**.
- In two dimensions, a second coordinate axis identical to the first but perpendicular to it is required; if we refer to this axis as the y axis, then any point on this axis is a y coordinate. This is also often called the **vertical axis**.
- In three dimensions, a third coordinate axis identical to the first two but perpendicular to both is required; if we refer to this axis as the z axis, then any point on this axis is a z coordinate. If the x and y axes are in the plane of the screen, then its unit vector $\hat{k}$ is directed out of the screen, but we represent the z axis as angled to make visualization easier.

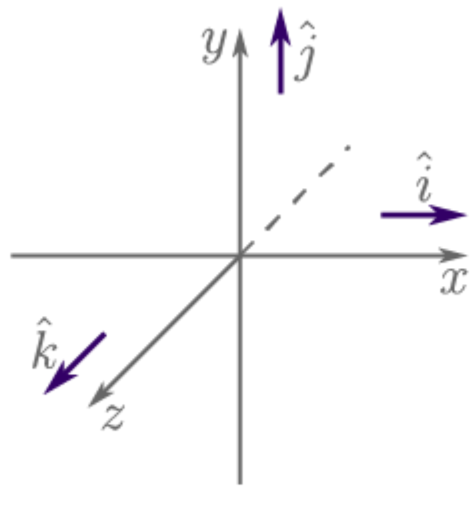
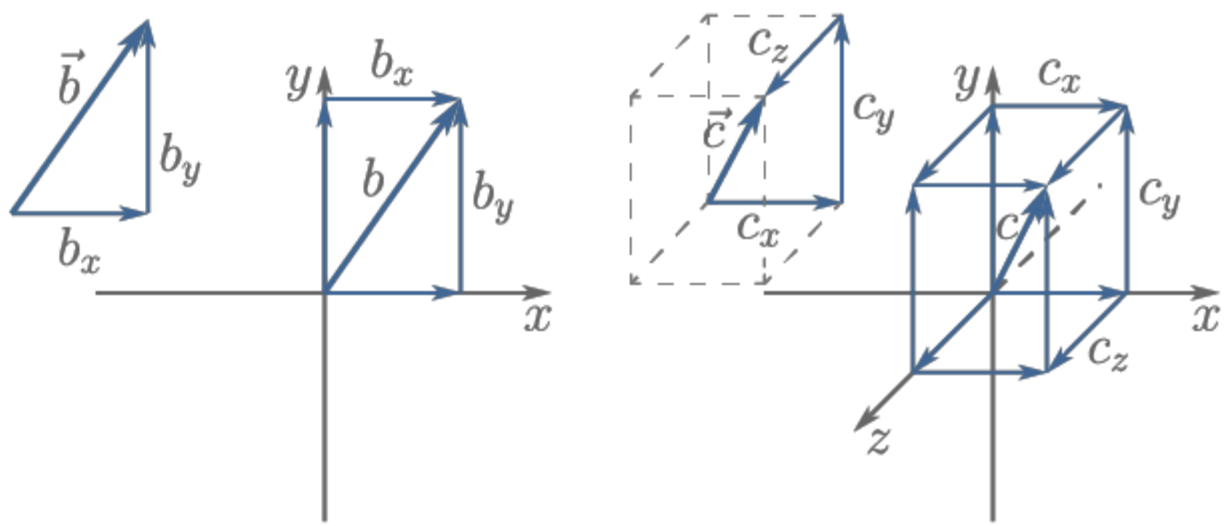


Figure 2.22 A three-dimensional Cartesian coordinate system is displayed. The arrow on each axis is

directed towards positive infinity. The corresponding unit vectors for the three axes are also shown.

Vector Components

The figure below shows two vectors. Vector $\vec{b}$, shown on the left in the figure below, is in the x-y plane. The vector starts at the origin and ends at a point with scalar coordinates (b_x, b_y). We can also see that vector $\vec{b}$ may be expressed as a sum of two vectors that are parallel to the coordinate axes and are known as the **vector components**. The inset to the left shows that $\vec{b}$ is sum of two vector components labeled $b_x \text{ and } b_y$. The figure on the right side of the figure shows vector $\vec{c}$, which has vector components in all three dimensions, $c_x, c_y, \text{ and } c_z$.



Vectors and their components have been added to each of the one-dimensional, two-dimensional, and three-dimensional coordinate systems.

Because the vector components are parallel to the coordinate axes, each may be expressed as the product of a scalar coefficient and a unit vector, which is called a **scalar component**. A scalar component has a positive value. If you see a minus sign in front of a component, that sign indicates the negative direction. Written in terms of scalar components, the vectors in the image are

$$\begin{aligned}\vec{b} &= b_x\,\hat{i} + b_y\,\hat{j} \\ \vec{c} &= c_x\,\hat{i} + c_y\,\hat{j} + c_z\,\hat{k}\end{aligned} \tag{2.31}$$

Vector Magnitude

Let's consider the magnitude of each vector. In one dimension, the length of a vector is the absolute value of its only component. For two and three dimensions, apply the Pythagorean theorem to the scalar components, which works because the axes are orthogonal. In other words, take the square root of the sum of the squared scalar components.

$$c = \sqrt{c_x^2 + c_y^2 + c_z^2}$$

The length of the vector may be given in terms of the coordinates of the head of the vector, which is the final point, and the tail, which is the initial point. If "f" indicates final and "i" indicates initial, then the length of a three-dimensional vector, $\vec{c}$, is given by

$$c = \sqrt{(x_\mathrm{f} - x_\mathrm{i})^2 + (y_\mathrm{f} - y_\mathrm{i})^2 + (z_\mathrm{f} - z_\mathrm{i})^2}$$

In the case of one and two dimensions, the terms that don't apply are excluded.

Polar Coordinates

In addition to Cartesian coordinates, plane polar coordinates will be very useful in certain situations. Consider Figure 2.7 below. The **pole** of the coordinate system is located at the center and is similar to the origin in the Cartesian coordinate system. The **radial coordinate**, r, of any point is the distance from the pole along a radial ray. The radial ray that is directed horizontally to the right from the pole is called the

polar axis. The **angular coordinate**, ϕ, also known as the **azimuth**, is measured counterclockwise from the polar axis. Note that points with a constant angular coordinate form a radial ray while points with a constant radial coordinate form a circle with the pole as the center. Angles are typically given in either degrees or radians.

In Cartesian coordinates, the orthogonal Cartesian unit vectors $\hat{\imath}$ and $\hat{\jmath}$ are directed in the same directions at all points in the plane. Plane polar coordinates also have a special pair of orthogonal unit vectors, but their directions change with the azimuthal angle, ϕ. At each point, the **radial unit vector**, $\hat{r}$, is directed away from the pole along a radial ray; the **azimuthal unit vector**, $\hat{\phi}$, is perpendicular to the radial unit vector and toward increasing angle.

Converting Cartesian Coordinates to Polar Coordinates

Consider a vector in Cartesian unit-vector notation as

$$\vec{f} = f_x\,\hat{\imath} + f_y\,\hat{\jmath}$$

Taking the origin as the pole and the positive x axis as the polar axis, the radial coordinate may be obtained using the Pythagorean theorem according to

$$r = \sqrt{f_x^2 + f_y^2}$$

The direction of the vector may be obtained using the inverse tangent function of the ratio of the magnitudes of the coordinates as

$$\phi = \tan^{-1}\left(\frac{f_y}{f_x}\right)$$